Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

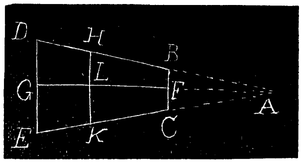
131. Proposed by **M. A. GRUBER. A. M.**, War Department, Washington, D. C.

A right frustum of a cone whose radii of the bases are r and s, $r>s$, is to be divided into n parts of equal volume by sections parallel to the bases. What are the altitudes of the respective parts?

Solution by the PROPOSER.

Let $BCED$=section of given frustum through the centers of the bases.

Produce DB and EC until they meet in A; and draw AG perpendicular to DE. Then AF and AG= the respective altitudes of cones ABC and ADE; FG =altitude of given frustum; $DG=r$, and $BF=s$.



Draw HK parallel to DE so that the frustum with altitude FL is m/n part of the entire frustum.

Put $a=FG$; and let $x=AF$, $y_m=AL$, and $z_m=HL$.

The similar triangles ABF and ADG give $x:x+a=s:r$; or $x:a=s:r-s$.

$$\therefore x=\frac{as}{r-s}, \text{ and } x+a=\frac{ar}{r-s}.$$

$$\therefore \frac{\pi ar^3}{3(r-s)}=\text{volume of cone } ADE;\ \frac{\pi as^3}{3(r-s)}=\text{volume of cone } ABC;$$

$\frac{1}{3}\pi a(r^2+s^2+rs)$=volume of given frustum $BCED$;

$\dfrac{\pi am(r^2+s^2+rs)}{3n}$=volume of frustum $BCKH$;

and $\dfrac{\pi a[mr^3+(n-m)s^3]}{3n(r-s)}$=volume of cone AHK=cone ABC+frustum $BCKH$.

From the similar volumes, cones ADE and AHK, we have

$$\frac{\pi ar^3}{3(r-s)} : \frac{\pi a[mr^3+(n-m)s^3]}{3n(r-s)}=\left(\frac{ar}{r-s}\right)^3 : y_m{}^3.$$

$$\therefore y_m=\frac{a}{n(r-s)}\sqrt[3]{n^2[mr^3+(n-m)s^3]},$$

$$\text{and } y_{m+1}=\frac{a}{n(r-s)}\sqrt[3]{n^2[(m+1)r^3+(n-m-1)s^3]}.$$

$$\therefore\ y_{m+1}-y_m=\frac{a}{n(r-s)}\{\sqrt[3]{n^2[(m+1)^3r^2+(n-m-1)s^3]}-\sqrt[3]{n^2[mr^3+(n-m)s^3]}\},$$

which is the general value for the respective altitudes of the n equal parts of the given frustum.

The limits of m are *zero* and n.

$$y_0=x=\frac{as}{r-s},\text{ and } y_n=x+a=\frac{ar}{r-s}.$$

Hence as the altitudes of the equal parts diminish from s to r, $y_1-y_0=$ the greatest altitude and $y_n-y_{n-1}=$the least altitude.

The radius of the mth section is $z_m=\frac{1}{n}\sqrt[3]{n^2[mr^3+(n-m)s^3]}$.

Put $a=12$, $r=3$, $s=2$, and $n=4$.

Then $y_{m+1}-y_m=6\{\sqrt[3]{[2(19m+51)]}-\sqrt[3]{[2(19m+32)]}\}$.

Whence $y_1-y_0=6[\sqrt[3]{(102)}-4]=4.034$; $y_2-y_1=6[\sqrt[3]{(140)}-\sqrt[3]{(102)}]=3.121$; $y_3-y_2=6[\sqrt[3]{(178)}-\sqrt[3]{(140)}]=2.596$; and $y_4-y_3=6[6-\sqrt[3]{(178)}]=2.249$.

Also $z_1=\frac{1}{2}\sqrt[3]{(102)}=2.336$; $z_2=\frac{1}{2}\sqrt[3]{(140)}=2.596$; and $z_3=\frac{1}{2}\sqrt[3]{(178)}=2.812$.

Also solved in a very excellent manner by *G. B. M. ZERR*, and *J. SCHEFFER*.

ALGEBRA.

---

107. Proposed by CHARLES E. MYERS, Canton, Ohio.

Given $xyz=18\ldots.(1)$; $x^2+y^2+z^2=33\ldots.(2)$; and $(x^2-yz)^3+(y^2-xz)^3+(z^2-xy)^3-3(x^2-yz)(y^2-xz)(z^2-xy)=6561\ldots.(3)$; to find x, y, and z.

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Expanding, uniting terms, and extracting square root of [3], we have

$$x^3+y^3+z^3-3xyz=81\ldots.[4].$$

Substituting $xyz=18$, and transposing, [4] becomes

$$x^3+y^3+z^3=135\ldots.[5].$$

Put $y=x+v$, and $z=x-v$.

Then, [1] becomes $x^3-xv^2=18\ldots.[6]$,

[2] becomes $3x^2+2v^2=33\ldots.[7]$,

and [5] becomes $3x^3+6xv^2=135\ \ldots[8]$.

From [6] and [8], we readily find $x=3$. Whence $v=\pm\sqrt{3}$.

$\therefore\ x=3$, $y=3\pm\sqrt{3}$, and $z=\mp\sqrt{3}$.